CSCE 551/MATH 562, Homework 2
due Monday 2/12/2024

Exercise 1.19: Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata.

b. $(((\texttt{00})^*(\texttt{11})) \cup \texttt{01})^*$

Exercise 1.21: Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.

b. [Given in tabular form (this is a DFA):]

	a	b
$\rightarrow *1$	2	2
2	2	3
$*3$	1	2

Exercise 1.29: Use the pumping lemma to show that the following languages are not regular.

b. $A_2 = \{www \mid w \in \{\texttt{a}, \texttt{b}\}^*\}$

c. $A_3 = \{\texttt{a}^{2^n} \mid n \geq 0\}$ (Here, $\texttt{a}^{2^n}$ means a string of 2^n a's.)

Problem 1.40: Recall that a string x is a ***prefix*** of string y if a string z exists where $xz = y$, and that x is a ***proper prefix*** of y if in addition $x \neq y$. In each of the following parts, we define an operation on a language A. Show that the class of regular languages is closed under that operation.

b. $\mathit{NOEXTEND}(A) = \{w \in A \mid w \text{ is not a proper prefix of any string in } A\}$. [Note: I have corrected the textbook's wording, changing "the" to "a" because a string can have many proper prefixes.]

Problem 1.43: Let A be any language. Define $\mathit{DROP\text{-}OUT}(A)$ to be the language containing all strings that can be obtained by removing one symbol from a string in A. Thus,

$$\mathit{DROP\text{-}OUT}(A) = \{xz \mid xyz \in A \text{ where } x, z \in \Sigma^*,\ y \in \Sigma\}\ .$$

Show that the class of regular languages is closed under the *DROP-OUT* operation. Give both a proof by picture and a more formal proof by construction as in Theorem 1.47.

Non-Textbook Exercise 1: Let $\Sigma := \{\mathtt{a}, \mathtt{b}\}$, and let L be the language of all strings $w \in \Sigma^*$ such that a $\mathtt{b}$ occurs somewhere in the *second* half of w, that is,

$$L := \{w \in \Sigma^* : (\exists t, u \in \Sigma^*)[\ w = t\mathtt{b}u \ \&\ |t| > |u|\ ]\} .$$

Show that L is not pumpable (hence not regular by the Pumping Lemma).

Non-Textbook Exercise 2: Let $\Sigma := \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. For any $w \in \Sigma^*$, let $\mathit{MoreA}(w)$ be the set of all possible strings obtained from w by replacing each occurrence of $\mathtt{a}$ in w with a string of one or more $\mathtt{a}$'s (not necessarily the same number for each occurrence). So for example,

$$\begin{aligned} \mathit{MoreA}(\mathtt{bc}) &= \{\mathtt{bc}\} , \\ \mathit{MoreA}(\mathtt{bac}) &= \{\mathtt{bac}, \mathtt{baac}, \mathtt{baaac}, \ldots\} , \\ \mathit{MoreA}(\mathtt{baca}) &= \{\mathtt{baca}, \mathtt{baaca}, \mathtt{bacaa}, \mathtt{baacaa}, \ldots\} . \end{aligned}$$

For any language $L \subseteq \Sigma^*$, define

$$\mathit{MoreA}(L) := \bigcup_{w \in L} \mathit{MoreA}(w) ,$$

that is, the result of applying $\mathit{MoreA}()$ to every string in L and collecting the resulting strings into a language.

Show that if L is regular, then $\mathit{MoreA}(L)$ is regular.