CSCE 551/MATH 562, Homework 1
due Monday 1/29/2024

The numbered exercises are from the textbook, written out for the purpose of comparing your book version's exercises with mine. (Note: "state diagram" is the same as "transition diagram.") You must do the exercises as worded below.

Exercise 1.4: Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

c. $\{w \mid w$ has an even number of $\mathtt{a}$'s and one or two $\mathtt{b}$'s$\}$

e. $\{w \mid w$ starts with an $\mathtt{a}$ and has at most one $\mathtt{b}\}$

Exercise 1.5: Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

d. $\{w \mid w$ is any string not in $\mathtt{a}^*\mathtt{b}^*\}$

f. $\{w \mid w$ is any string not in $\mathtt{a}^* \cup \mathtt{b}^*\}$

Exercise 1.6: Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{\mathtt{0}, \mathtt{1}\}$.

c. $\{w \mid w$ contains the substring $\mathtt{0101}$ (i.e., $w = x\mathtt{0101}y$ for some x and $y)\}$

l. $\{w \mid w$ contains an even number of $\mathtt{0}$'s or contains exactly two $\mathtt{1}$'s$\}$

Exercise 1.7: Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is $\{\mathtt{0}, \mathtt{1}\}$.

b. The language of Exercise 1.6c with five states

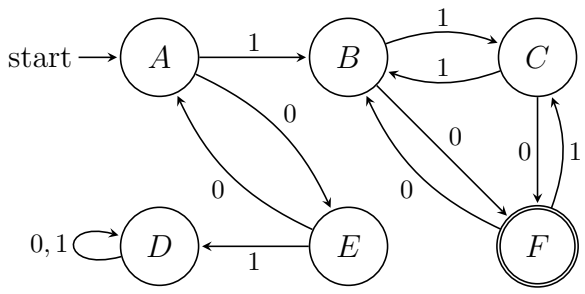
c. The language of Exercise 1.6l with six states

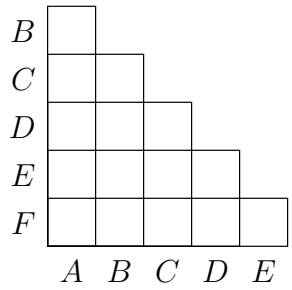
Exercise 1.16: Use the construction given in Theorem 1.39 to convert the following two nondeterministic finite automata to equivalent deterministic finite automata.

b. [Given in tabular form:]

	a	b	ε
$\rightarrow 1$	$\{3\}$	$\emptyset$	$\{2\}$
$*2$	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2,3\}$	$\emptyset$

Not in Textbook 1: Consider the DFA A (below left) over the alphabet $\{0,1\}$:





1. Fill in the distiguishability table to the right with X in each entry corresponding to a pair of distinguishable states.
2. Draw (as a transition diagram) the minimal DFA equivalent to A.

Not in Textbook 2: Consider the following DFA A (given in tabular form):

	0	1
$\rightarrow *q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

Show that $L(A)$ is the language of all binary representations of natural numbers that are multiples of 3. (Here we assume ε represents the number zero, which is a multiple of 3.) Hint: Prove the stronger statement that, for $k \in \{0,1,2\}$, the computation of A on input string w ends in state q_k iff w represents a number whose remainder is k when divided by 3. Make this argument by induction on $|w|$.