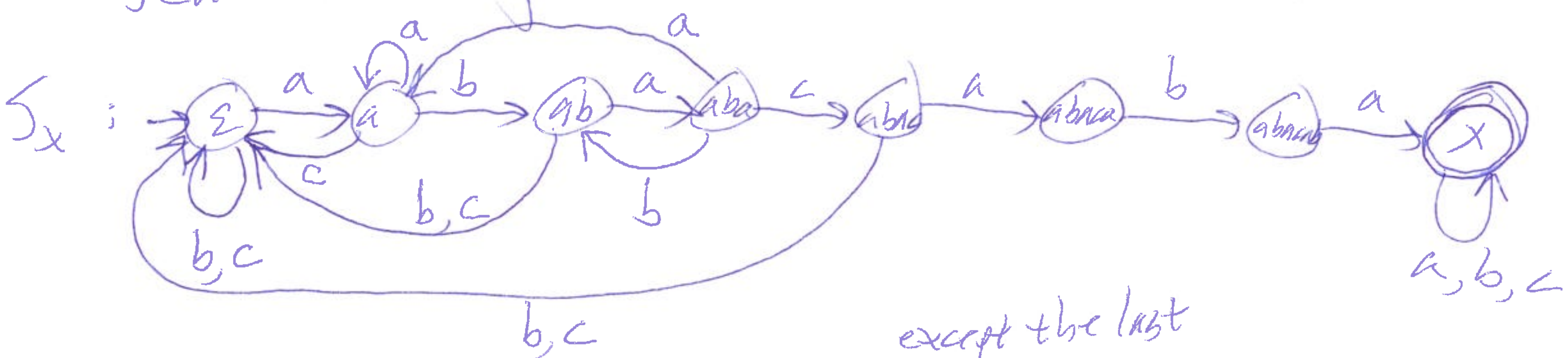
CSCE 355
1/22/2024

Text search DFA

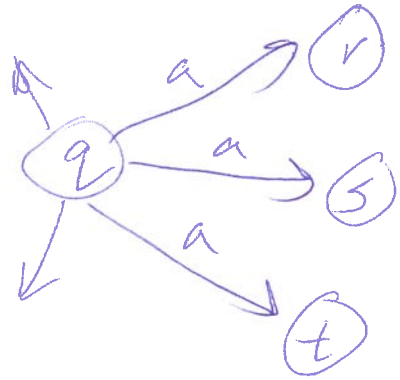
$\Sigma = \{a,b,c\}$

Search string $x = abacaba$, DFA S_x



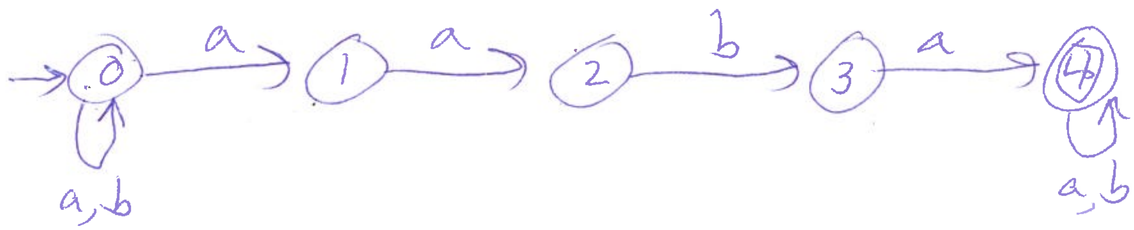
Ex: fill in the rest! Each state, except the last, reflects the longest prefix of the search string that is a suffix of the input read so far.

For an NFA (nondeterministic finite automaton) transition diagram is unrestricted: any number of arrows (incl. 0) leaving the same state with the same label.



NFA accepts if there is some sequence of states from the start state to an accepting state that reads the entire input.

Ex: $x = aaba$ NFA to search for x ($\Sigma = \{a,b\}$)

N: 

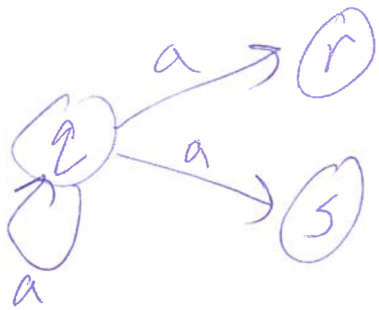
Input: $w = abaabaa$

step	read so far:	possible states
0	ε	0
1	a	0, 1
2	ab	0
3	aba	0, 1
4	abaa	0, 1, 2
5	abaab	0, 3
6	abaaba	0, 1, 4
7	abaabaa	0, 1, 2, 4 accepting

~~The~~ Definition: A nondeterministic finite automaton (NFA) is a 5-tuple $\langle Q, \Sigma, \delta, q_0, F \rangle$ where Q, Σ, q_0, F are just as with a DFA, and

$$\delta : Q \times \Sigma \to 2^Q$$

$\delta(q, a)$ is a set of states:



$$\delta(q, a) = \{q, r, s\}$$

Def: Given NFA $N := \langle Q, \Sigma, \delta, q_0, F \rangle$ and input $w \in \Sigma^*$, a computation (path) of N on input w is a sequence of states $s_0, s_1, \ldots, s_n$ such that, there exist symbols $w_1, w_2, \ldots, w_n \in \Sigma$ such that

1) $w = w_1 w_2 \cdots w_n$ $\quad (n = |w|)$
2) $s_0 = q_0$
3) For all i, $1 \leq i \leq n$,

$$s_i \in \delta(s_{i-1}, w_i)$$

Say the path ends in state s_n.

N accepts w if there exists a comp. path on input w that ends in an accepting state.

Notice: Every DFA can be converted (trivially) into an equivalent NFA:

DFA $\qquad\qquad$ NFA

$$\delta(q,a)=r \quad \Longrightarrow \quad \delta(q,a)=\{r\}$$

"equivalent" means recognizing the same lang.

Thm: For any NFA A there exists an equivalent DFA D (i.e., $L(D)=L(A)$).

Proof: "Sets of states" construction:

Given $A := \langle Q, \Sigma, \delta, q_0, F\rangle$ an NFA, we define a DFA

$$D := \langle 2^Q, \Sigma, \Delta, \{q_0\}, \mathcal{F}\rangle$$

where

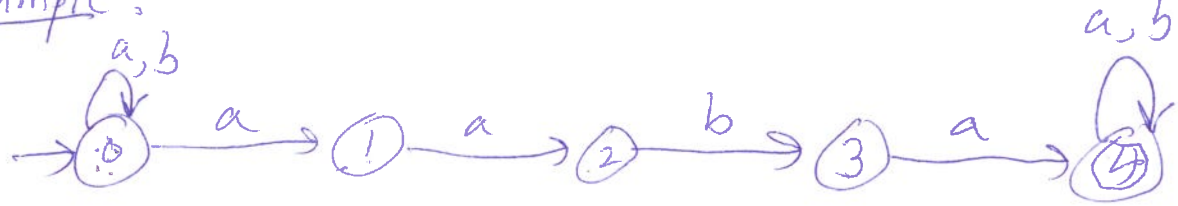
$$\mathcal{F} := \{S \subseteq Q : S \text{ contains an accept state of } A \; [S\cap F \neq \emptyset]\}$$

and for all $S \subseteq Q$ and $a \in \Sigma$,

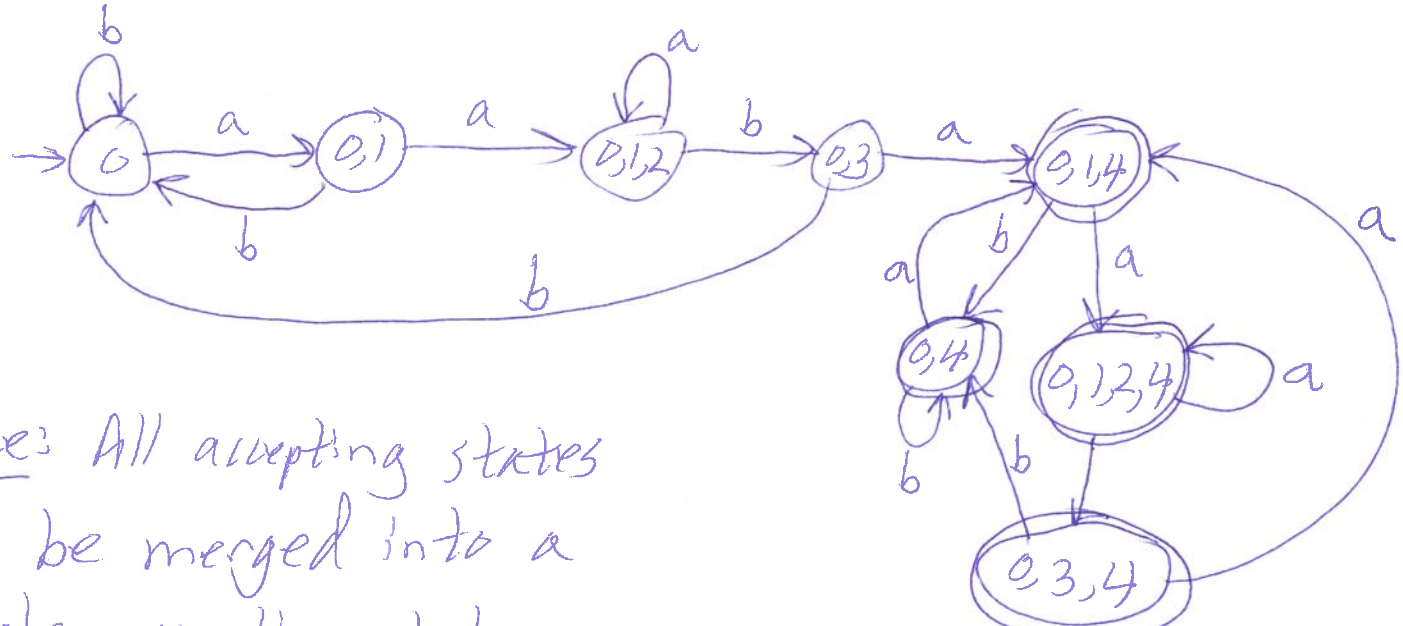
$$\Delta(S,a) := \bigcup_{q\in S} \delta(q,a).$$

Example:

A:



D:



Notice: All accepting states can be merged into a single accepting state with self-loops on a,b.

ε-transitions (ε-moves)

An ε-NFA is an NFA that allows transitions without consuming an input symbol:

$$q \xrightarrow{\varepsilon} r$$

Next

For every ε-NFA there exists an equivalent NFA (without ε-moves) with the same state set.