CSCE 355
3/21/22

① CFG $\longrightarrow$ PDA

$G \longmapsto P$ such that $N(P) = L(G)$

Given a CFG $G = \langle V, T, S, R \rangle$

nonterms, terminals, start symbol, productions

$P = \langle \{q\}, T, T \cup V, \delta, q, S, \emptyset \rangle$

where, for every $a \in T$ ~~$X \in T$~~ we have

$(q, \varepsilon) \in$ ~~$\delta(a, a)$~~ $\delta(q, a, \underline{a})$ "matching a"

and for every ~~$X \in T$ and production $X \rightarrow$~~
$A \in V$ and production $A \rightarrow \beta$ in R
we have

$(q, \beta) \in \delta(q, \underline{\underline{\varepsilon}}, A)$ "expanding A"

and nothing else in δ.

Ex: $E \rightarrow T T'$

$T' \rightarrow \varepsilon \mid + T T'$

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$$T \rightarrow FF'$$

$$F' \rightarrow \varepsilon \mid *FF'$$

$$F \rightarrow c \mid (E)$$

$$P = \langle \{q\}, \{c, +, *, '(', ')'\}, \{c, +, *, '(', ')', E, T, T', F, F'\}, \delta, E, \emptyset \rangle$$ and

$$\delta(q, c, c) = \{(q, \varepsilon)\}$$

$$\delta(q, +, +) = \{ \text{ " } \}$$

$$\delta(q, *, *) = \{ \text{ " } \}$$

$$\delta(q, '(', '(') = \{ \text{ " } \}$$

$$\delta(q, ')', ')') = \{ \text{ " } \}$$

$$\delta(q, \varepsilon, E) = \cancel{\{(q, FF'), (q, \dots)\}} \ \{(q, TT')\}$$

$$\delta(q, \varepsilon, T) = \{(q, FF')\}$$

$$\delta(q, \varepsilon, T') = \{(q, \varepsilon), (q, +TT')\}$$

$$\delta(q, \varepsilon, F) = \{(q, c), (q, '('E')')\}$$

③ $\delta(q, \varepsilon, F') = \{(q, \varepsilon), (q, *FF')\}$

Sample input: $c*(c+c)$

Leftmost derivation:

$$\underset{(0)}{\underline{E}} \Rightarrow \underset{(1)}{\underline{T}T'} \Rightarrow \underset{(2)}{\underline{F}F'T'} \Rightarrow \underset{(3)}{c\underline{F'}T'} \Rightarrow \underset{(4)}{c*\underline{F}F'T'}$$

$$\Rightarrow \underset{(5)}{c*(\underline{E})F'T'} \Rightarrow \underset{(6)}{c*(\underline{T}T')F'T'}$$

$$\Rightarrow \underset{(7)}{c*(\underline{F}F'T')F'T'} \Rightarrow \underset{(8)}{c*(c\underline{F'}T')F'T'}$$

$$\Rightarrow \underset{(9)}{c*(c\underline{T'})F'T'} \Rightarrow \underset{(10)}{c*(c+\underline{T}T')F'T'}$$

$$\Rightarrow \underset{(11)}{c*(c+\underline{F}F'T')F'T'} \Rightarrow \underset{(12)}{c*(c+c\underline{F'}T')F'T'}$$

$$\Rightarrow \underset{(13)}{c*(c+c\underline{T'})F'T'} \Rightarrow \underset{(14)}{c*(c+c)\underline{F'}T'}$$

$$\Rightarrow \underset{(15)}{c*(c+c)\underline{T'}} \Rightarrow \underset{(16)}{c*(c+c)}$$

Accepting computation of P on input $c*(c+c)$ [follows the leftmost derivation closely]:

$$(q, \overset{\varepsilon \text{ consumed}}{\underline{c*(c+c)}}, E) \vdash (q, \overset{\varepsilon}{c*(c+c)}, TT')$$

$$\vdash (q, \overset{\varepsilon}{c*(c+c)}, FF'T') \vdash (q, \overset{\varepsilon}{\underset{\uparrow}{c}*(c+c)}, \underset{\uparrow}{c}F'T')$$

$$\vdash (q, \overset{c}{*(c+c)}, F'T') \vdash (q, \overset{c}{*(c+c)}, *FF'T')$$

$$\vdash (q, \overset{c*}{(c+c)}, FF'T') \vdash (q, \overset{c*}{(c+c)}, (E)F'T')$$

$$\vdash (q, \overset{c*(}{c+c)}, (E)F'T') \vdash \text{etc}\ldots$$

$$\vdash (q, \overset{c*(c+c)}{\varepsilon}, \varepsilon)$$

Def: G CFG with vars V and terminals T, start symbol S. A sentential form is any string $\alpha \in (V \cup T)^*$ such that $S \overset{*}{\Rightarrow} \alpha$

Lemma: Given $G = \langle V, T, S, R \rangle$ as above, let P be the PDA constructed (as above) from G. ~~Let ID_0~~ For any $w \in T^*$ let

$ID_0 \vdash ID_1 \vdash \cdots \vdash ID_k$ be a computation path of P on input w (not necessarily complete).

(5) $ID_0 = (q, w, S)$

For each i, let x_i be the consumed portion of the input in ID_i. That is, if

$ID_i = (q, y_i, \alpha_i)$ then $w = x_i y_i$.

Then, for each i, $x_i \alpha_i$ is a sentential form in a leftmost derivation.

<u>Proof</u>: Induction on i.

$i = 0$: $x_0 = \varepsilon$ and $\alpha_0 = S$

so $x_0 \alpha_0 = S$ is the first sentential form in a leftmost derivation

Suppose true for ID_i that is $x_i \alpha_i$ is a sentential form in a L-M derivation and ID_{i+1} exists. ($ID_i \vdash ID_{i+1}$) WTS $x_{i+1} \alpha_{i+1}$ is also a sentential form. Since ID_{i+1} exists, $\alpha_i \neq \varepsilon$. So let

$\alpha_i = X\beta$ for some $X \in V \cup T$ and $\beta = (V \cup T)^*$.

<u>Case 1</u>: $X \in T$.

6 Only ~~leg~~ legal step is to match X, so

~~$ID_{i+1} \Rightarrow y_i$~~ $ID_i = (q, y_i, \alpha_i)$

$= (q, y_i, X\beta)$

$= (q, Xy, X\beta)$

(some $y \in V^*$)

Then $ID_{i+1} = (q, y, \beta)$ and

so $x_{i+1} = x_i X$ $\quad$ ($w = x_i y_i =$

$x_i X y = x_{i+1} y$)

But then, $\alpha_{i+1} = \beta$

$x_{i+1} \alpha_{i+1} = x_i X \beta = \underline{x_i \alpha_i}$ sentential form by ~~ind hyp.~~ inductive hypothesis.

Case 2: $X \in V$. Then only legal to expand on X.

So $x_{i+1} = x_i$ (expansion step is an ε-move)

(and $y_{i+1} = y_i$) and, $\alpha_{i+1} = \gamma\beta$ for some γ

7 such that $X \to \gamma$ is a production of G.

So $ID_{i+1} = (q, y_i, \gamma\beta)$ and

$$x_{i+1}\alpha_{i+1} = x_i \gamma \beta$$

$$\underbrace{x_i \alpha_i = x_i X \beta}_{\text{sentential form by assumption}}$$

But

$$x_i \underline{X} \beta \underset{\text{leftmost}}{\Rightarrow} x_i \gamma \beta$$

$\therefore$ $x_{i+1}\alpha_{i+1}$ is a sentential form in a leftmost derivation.

$\therefore$ Lemma proved by induction.

Cor: If ~~$w \in L(G)$~~ P accepts w then ~~P accepts w.~~ $w \in L(G)$