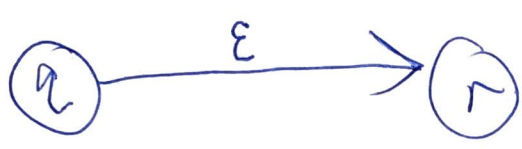


①

ϵ -transitions (ϵ -moves)



ϵ = empty string

Def.: An ϵ -NFA is tuple $\langle Q, \Sigma, \delta, q_0, F \rangle$
 where Q, Σ, q_0, F are same as for an NFA
 and

$$\delta : Q \times \left(\underbrace{\Sigma \cup \{\epsilon\}}_{\substack{\text{string of} \\ \text{length 0 or 1}}} \right) \rightarrow \cancel{2^Q}$$

In transition diagram:

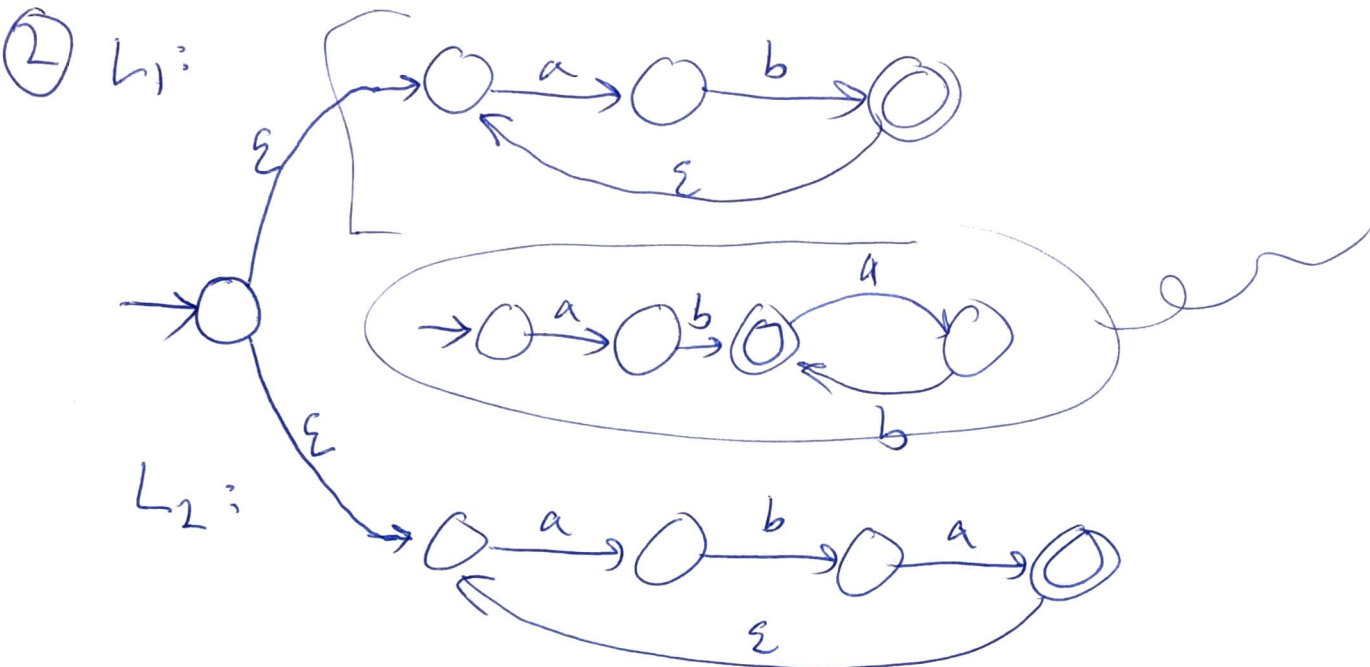


means $r \in \delta(q, \epsilon)$

$$\Sigma = \{a, b\}$$

$L = \{w \in \Sigma^+ : w \text{ is either one or more repetitions of } ab \text{ or one or more reps of } aba\}$

$L_0 = L_1 \cup L_2$ where $L_1 = \{ab, abab, ababab, \dots\}$
 $L_2 = \{aba, abanba, abaababnba, \dots\}$



Tabular form

	a	b	ϵ

Semantics

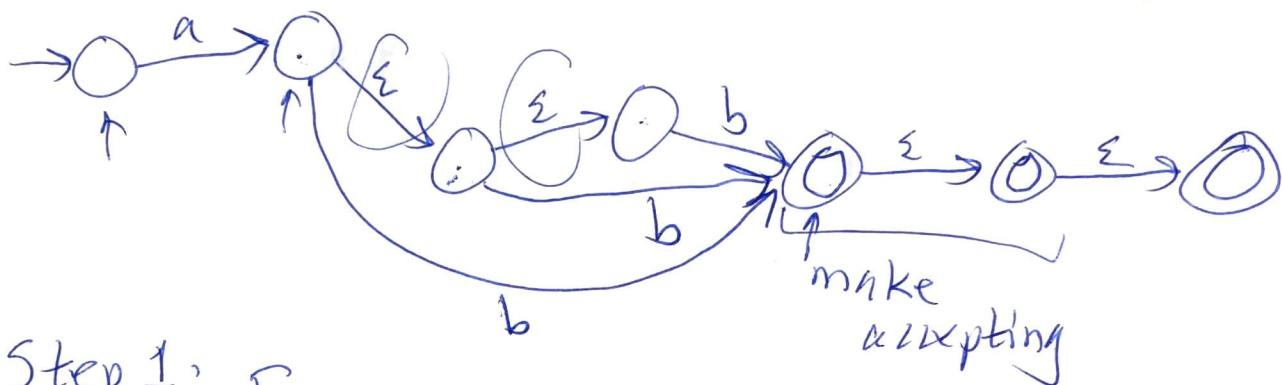
Same as for an NFA except
 $w = w_1 w_2 \dots w_n$ (each $w_i \in \Sigma \cup \{\epsilon\}$)
 (might be that $n > |w|$)

Ex: $w = abc = a\epsilon b = \epsilon\epsilon a\epsilon b\epsilon\epsilon = \dots$

Theorem: For any ϵ -NFA there exist an equivalent NFA with the state set and start state.

③ Idea: add non- ϵ -transitions to bypass the ϵ -transitions, rendering them unnecessary.

Proof: Given any ϵ -NFA $A = \langle Q, \Sigma, \delta, q_0, F \rangle$



Step 1: ~~For every~~

While there exist states $q, r \in Q$ such that

- $r \in F$
- $q \notin F$
- $r \in \delta(q, \epsilon)$



// make q accepting

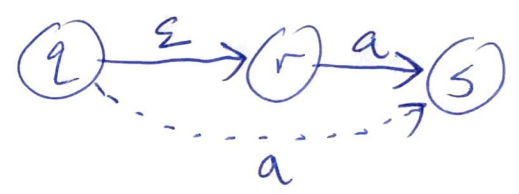
$F := F \cup \{q\}$

end-while

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Step 2: // make ϵ -moves unnecessary
// prior to ~~non~~ non- ϵ -moves

While there exist states $q, r, s \in Q$
(not necessarily distinct) ~~such that~~ and
there exists $a \in \Sigma$ such that



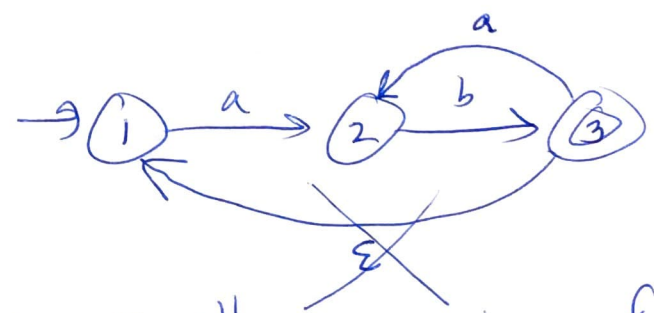
- $s \in \delta(r, a)$
- $r \in \delta(q, \epsilon)$
- $s \notin \delta(q, a)$

// Add s to $\delta(q, a)$

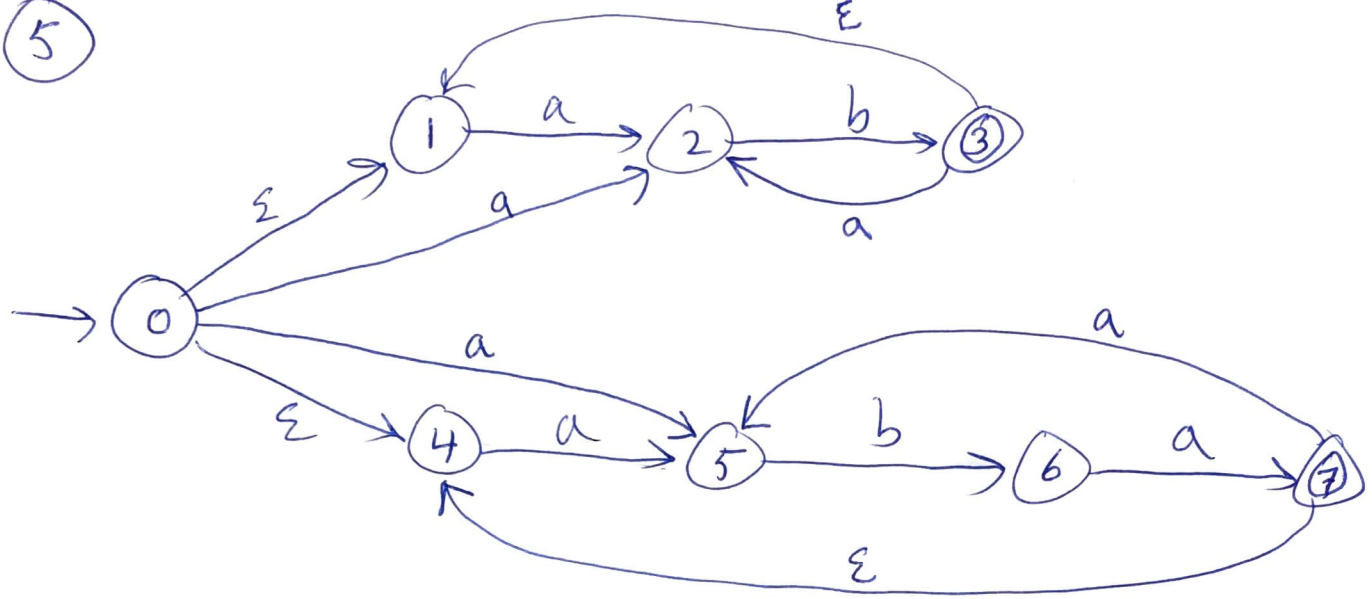
$$\delta(q, a) := \delta(q, a) \cup \{s\} \leftarrow$$

end-while

EX:



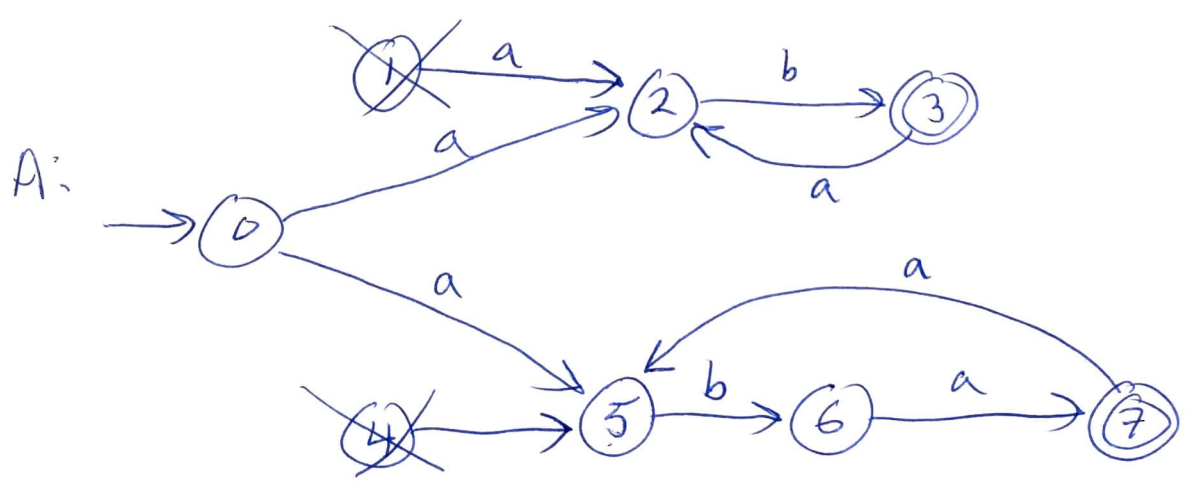
Step 3: Remove all ϵ -transitions from A :
 $\forall q, \delta(q, \epsilon) := \emptyset.$



Step 1: Nothing to do.

Step 2: add $3 \xrightarrow{a} 2$
 $7 \xrightarrow{a} 5$
 $0 \xrightarrow{a} 2$
 $0 \xrightarrow{a} 5$

Step 3: Remove ϵ -transitions;



Convert to a DFA;

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