CSCE 355

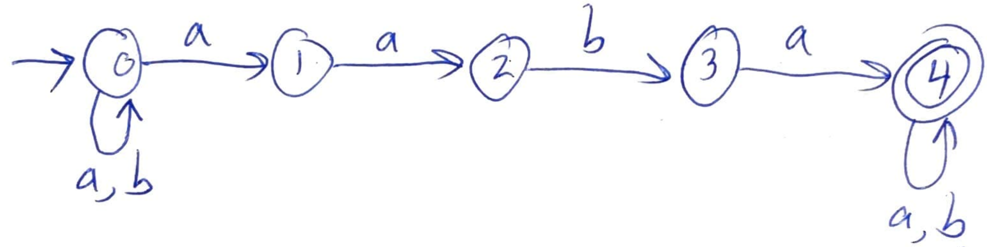
1/24/2022

NFAs (nondeterministic finite automata)

$L = \{w : w \text{ contains } aaba \text{ as a substring}\}$

$\Sigma = \{a, b\}$

A:

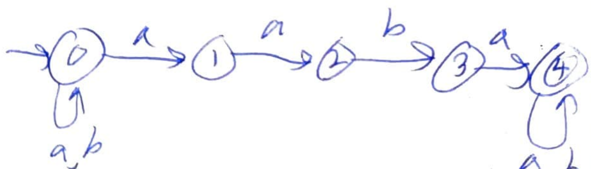


A accepts iff there is some choice of transitions that

1) ends in an accepting state, and

2) reads the entire input.

Simulating A



E. $w = abaabaa$

step	possible states	read so far
0	0	ε
1	0, 1	a
2	0	ab
3	0, 1	aba
4	0, 1, 2	abaa
5	0, 3	abaab

2

6	0,1,4	abaaba
7	0,1,2,4	abaabaa = w

accept

$w = ababb$

step	states possible	read so far
0	0	ε
1	0, 1	a
2	0	ab
3	0, 1	aba
4	0	abab
5	0	ababb = w

reject

Def: An NFA is a 5-tuple $\langle Q, \Sigma, \delta, q_0, F \rangle$ where Q, Σ, q_0, F are same as with a DFA, and

$$\delta : Q \times \Sigma \rightarrow 2^Q$$

[$2^Q = P(Q)$, the powerset of Q: set of all subsets of Q]

so $\forall q \in Q, a \in \Sigma$

$$\delta(q, a) \subseteq Q \quad \text{(set of states)}$$

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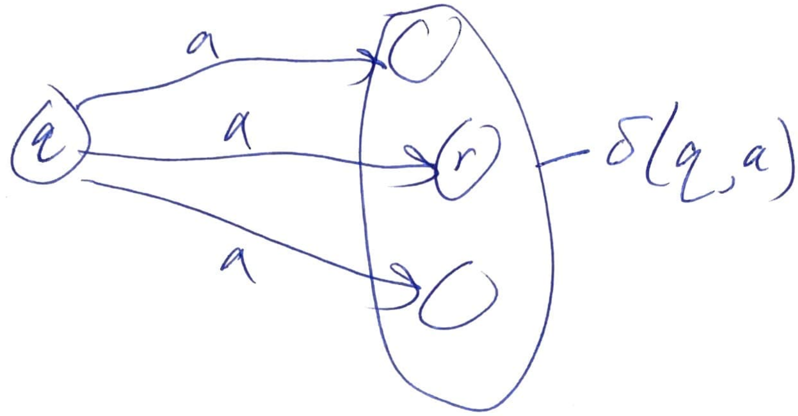
$\delta(q,a) = \{r : \text{there is an } a\text{-transition from } q \text{ to } r\}$



δ	a	b
$\rightarrow 0$	$\{0,1\}$	$\{0\}$
1	$\{2\}$	$\emptyset$
2	$\emptyset$	$\{3\}$
3	$\{4\}$	$\emptyset$
$*4$	$\{4\}$	$\{4\}$

Tabular form of A

$\emptyset$ = empty set = $\{\}$

Def: Let A be any NFA $(A = \langle Q, \Sigma, \delta, q_0, F \rangle)$ and let $w \in \Sigma^*$ be a string. Suppose $w = w_1 w_2 \cdots w_n$ $(w_i \in \Sigma)$

A (complete) computation path of A on input w is a sequence of states $s_0, s_1, \ldots, s_n \in Q$ such that

④

- $s_0 = q_0$
- For all i $(1 \le i \le n)$,

$$\underset{\substack{\uparrow \\ \text{member of}}}{s_i \in} \underbrace{\delta(s_{i-1}, w_i)}_{\text{set of states}}$$

Rmk [For a DFA, there is exactly one computation path on input w, but there could be any number of such paths for an NFA]

Say that <u>A accepts w</u> if there exists a complete comp path $s_0, \dots, s_n$ on input w such that $s_n \in F$.

[Define $L(A)$ as with DFAs:

$L(A)$ = lang. recognized by $A = \{w \in \Sigma^* : A \text{ accepts } w\}$]

<u>Theorem</u>: For every NFA, there is an equivalent ~~DA~~ DFA, that is, for any NFA A, there exists a DFA D such that $L(D) = L(A)$.

<u>Proof</u> (by construction):

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[Idea: states of D are sets of states of A. transitions are as with our simulation example.]

$$D := \langle 2^Q, \Sigma, \Delta, Q_0, \mathcal{F} \rangle$$

where

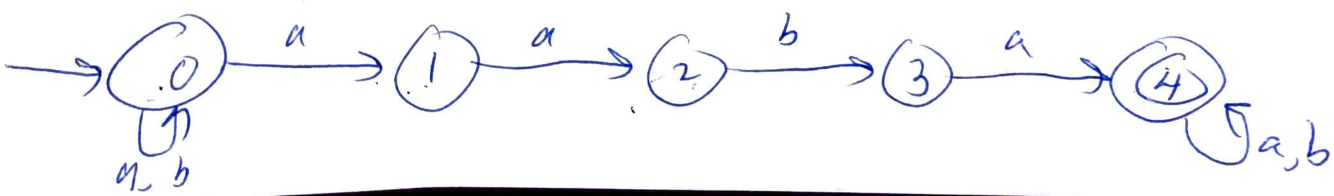
- $Q_0 = \{q_0\}$
- For every $S \in 2^Q$ $(S \subseteq Q)$ and every alphabet symbol $a \in \Sigma$, define

$$\Delta(S, a) = \bigcup_{q \in S} \delta(q, a)$$

- $\mathcal{F} := \{ S \subseteq Q : \underbrace{S \cap F \neq \emptyset}_{\text{S contains at least one accepting state of A.}} \}$

End of construction

Example A from before:



(6) Only need to include states reachable from Q_0