CSCE 355, Spring 2024, Assignment 6
Due April 15, 2024 at 11:30pm

The two exercises marked "optional" mostly build on one another and on previous exercises. The exercises should be considered in order.

1. Do Exercise 8.1.1(a). This has a solution on the book's website.

2. Do Exercise 8.2.4(a,b). This exercise has a complete solution on the book's website.

3. Let $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ be a Turing machine. As usual, we assume that $Q \cap \Gamma = \emptyset$. Recall that an *instantaneous description* (ID) of M is any string over the alphabet $Q \cup \Gamma$ containing exactly one symbol from Q. Give a DFA that recognizes the language of all IDs of M.

4. Let M be as in the last problem. Recall that if ID_1 and ID_2 are IDs of M, then $\mathrm{ID}_1 \vdash \mathrm{ID}_2$ means that ID_2 results from ID_1 by a single step of M. Let \$ be some symbol not in $Q \cup \Gamma$. The languages

$$L_1 := \{w\$x^R \mid w \text{ and } x \text{ are IDs of } M \text{ and } w \vdash x\}$$
$$L_2 := \{w^R\$x \mid w \text{ and } x \text{ are IDs of } M \text{ and } w \vdash x\}$$

are both context-free. (Recall that x^R and w^R are the reversals of strings x and w, respectively.) Describe how to build CFGs for L_1 and L_2, given a complete description of M.

5. (Optional) Let M and \$ be as in the last problem. Describe a PDA P that, given as input some string of the form $w\$x$, where w and x are IDs of M, accepts if and only if $w \not\vdash x$. In other words, P accepts iff it finds a "mistake" in M's transition from w to x. If you want, you may assume that the IDs w and x cover the exact same portion of M's tape, but this assumption is not necessary. You can stick to a high-level description of P instead of a formal one.

6. (Optional) Prove that there is no algorithm that decides, given a PDA P, whether P accepts all strings over its input alphabet. Hint: Given a TM M as in the last problem and an input string x, design a PDA P that accepts a string w if and only if w is *not* of the form $\mathrm{ID}_0\$\mathrm{ID}_1\$\cdots\$\mathrm{ID}_n$, where $\mathrm{ID}_0 \vdash \mathrm{ID}_1 \vdash \cdots \mathrm{ID}_n$ is a complete trace of a halting computation of M on input x. (P can be constructed using ideas from the last problem.) Then P accepts all strings if and only if M does not halt on input x. Conclude that any decision procedure for the former statement can be used to decide the latter, which we know is undecidable.